Pre-Calculus CP 1 – Section 8.5 **Name:** KEY

Matrix Application: Cryptography

Matrix operations, inverses, and determinants have several applications including performing a geometric transformation, solving a system, organizing data, and finding the area of a triangle. In this activity, we will explore another application, cryptography (or cryptology). A cryptogram (from the Greek word kryptos meaning "hidden") is a message written according to a secret code.

The 2x2 matrix method that you will be using to encode and decode a message is a simplified version of what was and is still actually used. During World War II, Navajo code talkers, 29 members of the Navajo Nation, developed a code based on the complex Navajo language that was used by US Armed Forces. They also used a rotor machine called the ECM Mark II, also known as SIGABA, to encode messages whose code was never broken! Another famous encoding machine was the ENIGMA machine. English mathematician, Alan Turning, was able to help break the code used by the German military. Many Hollywood movies such as "Sneakers" and "U571" explore the topic of encoding and decoding messages.

! = 27	C = 3	G = 7	K = 11	O = 15	S = 19	W = 23
___ = 0	D = 4	H = 8	L = 12	P = 16	T = 20	X = 24
A = 1	E = 5	I = 9	M = 13	Q = 17	U = 21	Y = 25
B = 2	F = 6	J = 10	N = 14	R = 18	V = 22	Z = 26

Example: What is Beethoven's favorite fruit? Coding matrix: $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$

Answer: 1 3 4 11 16 47 16 47 3 8 3 8 55 138

Set up your answer matrix like this: $\begin{bmatrix} 1 & 3 \\ 4 & 11 \\ 16 & 47 \\ 16 & 47 \\ 3 & 8 \\ 3 & 8 \\ 55 & 138 \end{bmatrix}$

Now multiply the answer matrix by the INVERSE of the original coding matrix, and then transfer your numbers back into letters. You should then have a good chuckle!!

Find the inverse using the 2x2 shortcut that involves determinants:

$$\text{If } A=\begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ then } A^{-1}=\frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}^{-1} = \frac{1}{5-6}\begin{bmatrix} 5 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}$$

Now multiply the inverse by the answer matrix. The numbers you get now have to be decoded and made into a phrase that answers the original question. If your output is not an integer between 0 and 26 then you made a mistake.

Matrix after performing decoding:

$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 14 & 1 \\ 14 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 27 \end{bmatrix}$$

A_BANAAAAAA!

What is Beethoven's favorite fruit?

Decoded Answer: _____

Riddle: What's a monster's favorite bean?

Secret Coded Answer: 5 7 82 119 67 94 70 98 20 29 33 49 135 189

Original* Coding Matrix:** $\begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$ => ***Inverse Matrix: $\begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}$

$\frac{1}{-15+14}\begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} -3 & 7 \\ 2 & -5 \end{bmatrix}$

Show your steps to find the inverse, and write the matrices you are multiplying below. You may multiply by hand or in your calculator. Write the answer matrix, and then decode using the chart from the notes to find answer to the riddle!

Matrix to decode multiplied by the inverse:

$$\begin{bmatrix} 5 & 7 \\ 82 & 119 \\ 67 & 94 \\ 70 & 98 \\ 20 & 29 \\ 33 & 49 \\ 135 & 189 \end{bmatrix} \times \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 8 & 21 \\ 13 & 1 \\ 14 & 0 \\ 2 & 5 \\ 1 & 14 \\ 21 & 0 \end{bmatrix}$$

Answer: A_HUMAN_BEAN!

Riddle: **I have a tail, and I have a head. I am not a snake. What am I?**

Secret Coded Answer: 91 254 57 158 11 33 50 141 16 44 66 183 3 9 69

192 83 235

Original Coding Matrix: $\begin{bmatrix} 4 & 11 \\ 1 & 3 \end{bmatrix}$ => ***Inverse Matrix:*** $\begin{bmatrix} 3 & -11 \\ -1 & 4 \end{bmatrix}$

Show your steps to find the inverse, and write the matrices you are multiplying below. You may multiply by hand or in your calculator. Write the answer matrix, and then decode using the chart from the notes to find answer to the riddle!

Matrix to decode multiplied by the inverse:

$$\begin{bmatrix} 91 & 254 \\ 57 & 158 \\ 11 & 33 \\ 50 & 141 \\ 16 & 44 \\ 66 & 183 \\ 3 & 9 \\ 69 & 192 \\ 83 & 235 \end{bmatrix} \begin{bmatrix} 3 & -11 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 19 & 15 \\ 13 & 5 \\ 0 & 11 \\ 9 & 14 \\ 4 & 0 \\ 15 & 6 \\ 0 & 3 \\ 15 & 9 \\ 14 & 27 \end{bmatrix}$$

Answer: SOME-KIND-OF-COIN!

Riddle: **What is the location of the lakes in the team name "Los Angeles Lakers?"**

Secret Coded Answer: -17 -21 26 39 -17 -21 -60 -83 -65 -88 -98 -137 -135 -189

Original Coding Matrix: $\begin{bmatrix} -5 & -7 \\ 2 & 3 \end{bmatrix}$ => ***Inverse Matrix:*** $\begin{bmatrix} -3 & -7 \\ 2 & 5 \end{bmatrix}$

$\frac{1}{-15+14}\begin{bmatrix} 3 & 7 \\ -2 & -5 \end{bmatrix} =$

Show your steps to find the inverse, and write the matrices you are multiplying below. You may multiply by hand or in your calculator. Write the answer matrix, and then decode using the chart from the notes to find answer to the riddle!

Matrix to decode multiplied by the inverse:

$$\begin{bmatrix} -17 & -21 \\ 26 & 39 \\ -17 & -21 \\ -60 & -83 \\ -65 & -88 \\ -98 & -137 \\ -135 & -189 \end{bmatrix} \times \begin{bmatrix} -3 & -7 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 9 & 14 \\ 0 & 13 \\ 9 & 14 \\ 14 & 5 \\ 19 & 15 \\ 20 & 1 \\ 27 & 0 \end{bmatrix}$$

Answer: IN - MINNESOTA!

Riddle: **Not alive, but I grow… No lungs, but I need air…**

No mouth, but water kills me… What am I?

Secret Coded Answer: 171 -54 58 -19 3 -1 18 -6 231 -73 262 -83 513 -162

Original Coding Matrix: $\begin{bmatrix} 19 & -6 \\ 3 & -1 \end{bmatrix}$ => ***Inverse Matrix***: $\begin{bmatrix} 1 & -6 \\ 3 & -19 \end{bmatrix}$

Show your steps to find the inverse, and write the matrices you are multiplying below. You may multiply by hand or in your calculator. Write the answer matrix, and then decode using the chart from the notes to find answer to the riddle!

Matrix to decode multiplied by the inverse:

$$\begin{bmatrix} 171 & -54 \\ 58 & -19 \\ 3 & -1 \\ 18 & -6 \\ 231 & -73 \\ 262 & -83 \\ 513 & -162 \end{bmatrix} \times \begin{bmatrix} 1 & -6 \\ 3 & -19 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 1 & 13 \\ 0 & 1 \\ 0 & 6 \\ 12 & 1 \\ 13 & 5 \\ 27 & 0 \end{bmatrix}$$

Answer: I-AM-A-FLAME!-

Riddle: **I never stop, but never move away. Have neither lungs nor throat, but still I make a mighty roar. What am I?**

Secret Coded Answer: 45 63 -21 -32 -2 -3 -46 -69 -35 -53 -11 -19 28 39 36 48 135 189

Original Coding Matrix: $\begin{bmatrix} 5 & 7 \\ -2 & -3 \end{bmatrix}$ => ***Inverse Matrix***: $\begin{bmatrix} 3 & 7 \\ -2 & -5 \end{bmatrix}$

Show your steps to find the inverse, and write the matrices you are multiplying below. You may multiply by hand or in your calculator. Write the answer matrix, and then decode using the chart from the notes to find answer to the riddle!

Matrix to decode multiplied by the inverse:

$$\begin{bmatrix} 45 & 63 \\ -21 & -32 \\ -2 & -3 \\ -46 & -69 \\ -35 & -53 \\ -11 & -19 \\ 28 & 39 \\ 36 & 48 \\ 135 & 189 \end{bmatrix} \times \begin{bmatrix} 3 & 7 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 1 & 13 \\ 0 & 1 \\ 0 & 23 \\ 1 & 20 \\ 5 & 18 \\ 6 & 1 \\ 12 & 12 \\ 27 & 0 \end{bmatrix}$$

Answer: I-AM-A-WATERFALL!

Riddle: **I pass before the sun, yet make no shadow. What am I?**

Secret Coded Answer: -36 -99 9 28 20 60 -27 -73 23 69 -22 -57 11 37

<u>Original</u> Coding Matrix: $\begin{bmatrix} -4 & -11 \\ 1 & 3 \end{bmatrix}$ => ***Inverse Matrix:*** $\begin{bmatrix} -3 & -11 \\ 1 & 4 \end{bmatrix}$

$\frac{1}{-12--11}\begin{bmatrix} 3 & -11 \\ -1 & -4 \end{bmatrix}$

Show your steps to find the inverse, and write the matrices you are multiplying below. You may multiply by hand or in your calculator. Write the answer matrix, and then decode using the chart from the notes to find answer to the riddle!

Matrix to decode multiplied by the inverse:

$$\begin{bmatrix} -36 & -99 \\ 9 & 28 \\ 20 & 60 \\ -27 & -73 \\ 23 & 69 \\ -22 & -57 \\ 11 & 37 \end{bmatrix} \times \begin{bmatrix} -3 & -11 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 1 & 13 \\ 0 & 20 \\ 8 & 5 \\ 0 & 23 \\ 9 & 14 \\ 4 & 27 \end{bmatrix}$$

Answer: I_AM_THE_WIND!